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A NEW CALIBRATION OF THE EXTRAGALACTIC DISTANCE SCALE USING CEPHEIDS AND RR LYRAE STARS

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ABSTRACT

Visual absolute magnitudes of classical Cepheids, metal-poor RR Lyrae stars, and short-period type II Cepheids have been determined with very high precision by combining a large number of old and new astrophysical data. Five independent methods (four of them observational and one theoretical) have been successfully used: (1) secular and statistical parallaxes; (2) moving-group parallaxes; (3) cluster main-sequence fitting; (4) the Baade-Wesselink method and its modifications; and (5) light-curve and velocity-curve fitting (theoretical method). It is shown that none of the methods depends for its validity on the (very uncertain) Hyades cluster distance modulus. The following results are obtained: for classical Cepheids, $\langle M_V \rangle(0.8) = -3.62$ based on $\langle M_V \rangle = -1.21\text{–}3.01 \log P$; for metal-poor RR Lyrae stars, $\langle M_V \rangle = 0.61$; and for normal 1–3 day type II Cepheids, $\langle M_V \rangle = -0.47$. The mean error in each case is ± 0.15 mag as estimated from the scatter among *all* the determinations used for each class of variable star, or ± 0.10 mag as derived from the scatter among the *means* of the determinations based on the various methods used for each class of variable star. These estimates include, therefore, both accidental and systematic errors.

External as well as internal consistency of the absolute magnitudes has been checked by intercomparing the results based on (*a*) the four independent *absolute* observational methods; (*b*) the independent *differential* data available for stars in the Large Magellanic Cloud, Small Magellanic Cloud, and galactic globular clusters; and (*c*) the independently generated *theoretical* data. Only one significant discrepancy emerged: the theoretically predicted absolute magnitudes for classical Cepheids are too bright by 0.5 mag (a 4 σ discrepancy), which is possibly a result of nonuniqueness of the light-curve and velocity-curve fitting technique for these stars, but is surely also connected with their well-known "mass discrepancy." It is found that the empirical period-luminosity relation for classical Cepheids is probably universal, i.e., insensitive to chemical composition, and that it is now well established from the large number of Cepheid members of galactic star groups.

Since the zero point of the distance scale has been uniquely determined, classical Cepheids and RR Lyrae stars now provide essentially identical distances to nearby galaxies: to the Large Magellanic Cloud, $(m - M)_0 = 18.5 \pm 0.1$; to the Small Magellanic Cloud, $(m - M)_0 = 18.8 \pm 0.1$; and to the center of our Galaxy, $R_0 = 8.6 \pm 0.5$ kpc. The major uncertainty in these values lies in the correction for interstellar extinction.

Subject headings: cosmology — stars: Cepheids — stars: RR Lyrae

I. INTRODUCTION

The extragalactic distance scale rests on a few fundamental calibrating objects, of which two, classical Cepheids and RR Lyrae stars, still retain their traditional superiority (Sandage and Tammann 1971; van den Bergh 1977*a*; de Vaucouleurs 1978*a*). Since these fundamental calibrators are used to calibrate brighter and larger objects, including the Galaxy itself, it is important to determine their absolute magnitudes as precisely as possible and (what is often ignored) to decide whether the best estimates for their absolute magnitudes are *mutually consistent*. Only then can the zero point of the distance scale be regarded as uniquely established.

Half a decade has elapsed since the last extensive review of this important subject. In the meantime, much new material has accumulated, and a search of the earlier literature reveals that both van den Bergh (1977*a*) and de Vaucouleurs (1978*a*) ignored a considerable amount of useful older material. The present effort is directed toward a redetermination of the absolute magnitudes of classical Cepheids, short-period type II Cepheids, and RR Lyrae stars by using the four traditional observational methods and a new theoretical method. The four observational methods include: (1) secular and statistical mean parallaxes of the variables; (2) moving-group parallaxes; (3) fitting of cluster main sequences to an adopted zero-age main sequence (ZAMS); and (4) the Baade-Wesselink method or its various modifications. Until very recently, no basically new approach to this problem had been devised since the work of Baade (1926). In the past two years, however, theory has been able to come to the aid of observation by providing fairly accurately computed light curves which can reveal the intrinsic luminosity of any RR Lyrae star or normal short-period type II Cepheid whose light curve is well measured, with *no* further observational information needed (Stothers 1981). Previous attempts at light-curve matching (e.g., Christy 1966) had to make use of other observational sources of luminosity or effective temperature. Although Eddington's (1918) much older method employed simply the period-mass-radius relation, it required observational knowledge of the mass and effective temperature of the star. Certainly a unique advantage of our new approach is its avoidance of any form of absolute geometric measurement, which is always a difficult problem in astronomy.

With these five adopted methods, the zero point of the galactic and extragalactic distance scale can be set on a relatively firm and self-consistent basis. As examples of applications, the distances to the galactic center and to the Large and Small Magellanic Clouds will be redetermined with a higher reliability than before.

II. OBSERVATIONAL CALIBRATIONS

a) *Classical Cepheids*

Visual absolute magnitudes of Cepheids and RR Lyrae stars will be quoted in this paper as intensity-means (which are equal to the equilibrium visual absolute magnitudes in the absence of variability). In the special case of classical Cepheids, $\langle M_V \rangle$ will be referred to a standard period, log P_0(days) = 0.8, with the help of published mean period-luminosity relations. Table 1 lists the recently published values of $\langle M_V \rangle$(0.8). No attempt has been made to adjust these values to a uniform system, since too many different assumptions went into each determination. It may be hoped that the unknown true errors associated with these values distribute normally and disappear in the mean of a large number of independent determinations.

It has been possible to enlarge significantly the number of stars N used in two of the traditional methods and to obtain new estimates of $\langle M_V \rangle$(0.8). These estimates are also entered in Table 1.

In the case of the Baade-Wesselink method, we have been able to use mean luminosities for 53 classical Cepheids listed by Cox (1979) in his Table 5. Cox searched the published literature for Wesselink radii and $\langle B \rangle - \langle V \rangle$ colors of Cepheids, which he then unreddened and converted to effective temperature, knowing that $\langle B_0 \rangle - \langle V_0 \rangle$ ought to be close to the equilibrium value for the unvarying star (Cox and Davis 1975; Davis and Cox 1980). Thus he was able to compute $L = 4\pi R^2 \sigma T_e$ for each star. When more than one luminosity was derived by Cox for any Cepheid, we have taken a straight average of his listed values. A least squares fit to his data yields $\langle M_{\text{bol}} \rangle = -2.04(\pm 0.15) - 2.18(\pm 0.16) \log P$, the standard deviation for a single star being ± 0.34 mag. From this we infer $\langle M_V \rangle(0.8) = -3.78$, since the bolometric correction for $\langle T_e \rangle = 5750$ K is very close to zero (Flower 1977).

The second case where we can improve the classical Cepheid calibration is the cluster main-sequence fitting method. Nearly twice as many Cepheids are now known to exist in galactic groups (viz., binary systems, clusters, and associations) as were known 6 years ago (van den Bergh 1977*a*); therefore, a new solution seems worthwhile. Probable group members are listed in Table 2. Although in no instance can group membership be deemed absolutely certain, we have felt it appropriate to reject SU Cas (Schmidt 1978), DW Per (Eggen 1965), AQ Pup (Turner 1981*a*), l Car (Eggen 1977, 1980; Schmidt 1980*a*), and RS Pup (Eggen 1977) for the reasons given by the authors cited. Van den Bergh (1977*a*) has

TABLE 1

OBSERVATIONAL DETERMINATIONS OF THE CLASSICAL CEPHEID $\langle M_V \rangle$ ZERO POINT

Method	$\langle M_V \rangle$ at log P_0 = 0.8	N	References
Secular or statistical parallax	-3.2 ± 0.3	18	Kraft and Schmidt 1963
	-3.91 ± 0.45	94	Geyer 1970
	-3.34 ± 0.40	33	Jung 1970
	-3.5 ± 0.4	45	Wielen 1974
	-3.7 ± 0.3	45	Clube and Dawe 1980
Moving group	-3.8 ± 0.2[a]	1	Eggen 1977
Modified Baade-Wesselink	-3.57 ± 0.14	5	Kraft 1961
	-3.63 ± 0.23	54	Fernie 1967
	-3.65 ± 0.15	9	de Vaucouleurs 1978*a* (data of Barnes *et al.* 1977)
	-3.78 ± 0.34	53	Present paper (data of Cox 1979)
Cluster main-sequence fitting[b]	-3.70 ± 0.06	5	Kraft 1961
	-3.64	9	Sandage and Tammann 1968
	-3.72	13	Sandage and Tammann 1969, 1971
	-3.50	14	van den Bergh 1977*a*
	-3.59 ± 0.18	26	Present paper
Unweighted mean	-3.62	...	

[a] Based on Eggen's $\langle M_V \rangle = -3.2 \pm 0.2$ for α UMi and $d\langle M_V \rangle/d \log P = -3.01$.
[b] Fitted to the standard ZAMS.

TABLE 2

CLASSICAL CEPHEIDS IN GALACTIC STAR GROUPS

Variable	P (days)	Group	$\langle M_V \rangle$	References
EV Sct	3.09	NGC 6664	−2.62	Sandage and Tammann 1969
α UMi	3.97	α UMi AB	−3.19	Turner 1977*c*
CE Cas b	4.48	NGC 7790	−3.20	Sandage and Tammann 1969
CF Cas	4.87	NGC 7790	−3.08	Sandage and Tammann 1969
CE Cas a	5.14	NGC 7790	−3.28	Sandage and Tammann 1969
UY Per	5.37	Ki 4 or Cz 8	−3.52	Turner 1977*b*
CV Mon	5.38	Anon.	−3.25	Turner 1976
VY Per	5.53	Per OB1	−3.91	Sandage and Tammann 1969
CS Vel	5.90	Ru 79	−3.05	Harris and van den Bergh 1976
V367 Sct	6.29	NGC 6649	−3.58	Turner 1981*b*
U Sgr	6.74	M25	−3.90	van den Bergh 1978
DL Cas	8.00	NGC 129	−3.84	Sandage and Tammann 1969
S Nor	9.75	NGC 6087	−4.03	Sandage and Tammann 1969
TW Nor	10.8	Lyngå 6	−4.00	Thé 1977 (mean modulus used)
VX Per	10.9	Per OB1	−4.34	Sandage and Tammann 1969
SZ Cas	13.6	Per OB1	−4.71	Sandage and Tammann 1969
VY Car	18.9	Car OB2	−4.95	Turner 1977*a*, 1978
RU Sct	19.7	Tr 35	−5.19	Turner 1980*b*
RZ Vel	20.4	Vel OB1	−5.12	Turner 1979*a*
VZ Pup	23.2	Anon.	−5.30	Havlen 1978
SW Vel	23.5	Anon.	−5.21	Turner 1979*a*
T Mon	27.0	Mon OB2	−5.55	Turner 1976
KQ Sco	28.7	Anon.	−5.52	Turner 1979*a*
SV Vul	45.0	Vul OB1	−6.00	Turner, in van den Bergh 1977*a*
GY Sge	51.0	Anon.	−6.34	Forbes 1982
S Vul	67.0	Vul OB2	−6.89	Turner 1980*a*

also rejected UY Per, VX Per, VY Per, and SZ Cas in the Per OB1 association, but Eggen's (1965) and Turner's (1977*b*) arguments persuade us to retain them. To achieve some measure of consistency, absolute magnitudes based only on *UBV* photometry, MK spectroscopy, and the standard ZAMS for B stars will be adopted here; Table 2 lists the absolute magnitudes. Extinction corrections (except for RZ Vel, VZ Pup, SW Vel, and KQ Sco) were taken to be those of neighboring B stars. Plotted against period in Figure 1, the absolute magnitudes cluster closely about the regression line

$$\langle M_V \rangle = \underset{\pm 0.11}{-1.18} - \underset{\pm 0.10}{3.01} \log P \, . \qquad (1)$$

The standard deviation for a single Cepheid is ± 0.18 mag.

Some of this scatter, however, is undoubtedly physically real, because at a given luminosity there must be a spread of colors corresponding to the local width of the instability strip, thus leading to a small spread of periods (Sandage 1958). The decreasing scatter above $P = 11$ days probably arises from a selection effect, namely, that long-period Cepheids have been assigned to groups (generally ill-defined associations and very small clusters) on the basis of absolute magnitudes already assumed from a mean period-luminosity relation. But the bias is actually immaterial, since the zero point is fixed mostly by the short-period Cepheids, whose group

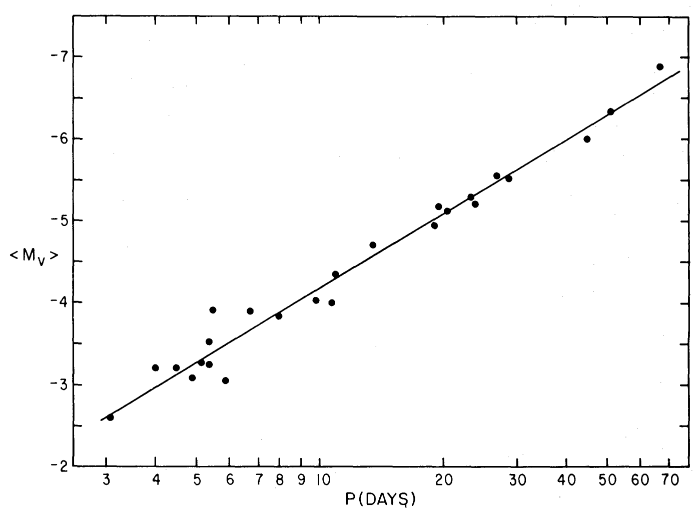

FIG. 1.—Period-luminosity relation for classical Cepheids belonging to galactic star groups

memberships are fairly secure, while the slope can be checked by observations of Cepheids in other galaxies. Photoelectric observations of Large Magellanic Cloud Cepheids indicate, in fact, a slope of -2.90 (Gascoigne and Shobbrook 1978) or -2.79 ± 0.20 (Martin, Warren, and Feast 1979), which agree well with our results. Known chemical composition differences among the Cepheids in the different galaxies should, according to Gascoigne (1974), introduce relatively little scatter into a mean period-luminosity relation (although not into a mean period-luminosity-color relation; see also Stift 1982 for physical arguments against using this more complicated kind of relation). Since our mean period-luminosity relation essentially confirms van den Bergh's (1977*a*) relation, it is possible to have confidence now in at least its *slope*. Our zero point, $\langle M_V \rangle(0.8) = -3.59 \pm 0.18$, is entered in Table 1.

The zero points of all the mean period-luminosity relations used for Table 1, however, rest *formally* on a Hyades distance modulus of $(m - M)_0 = 3.03$ obtained by van Bueren (1952) from an early proper-motion study of this cluster. More recent studies have incorporated new proper motions, as well as trigonometric, dynamical, and spectroscopic parallaxes, and have indicated the possible need for an increase by as much as $\delta(m - M)_0 \approx 0.27$ mag (de Vaucouleurs 1978*a*; Hanson 1980) or even $\delta(m - M)_0 \approx 0.40$ mag (Hanson 1975; McClure 1982; Egret, Keenan, and Heck 1982). It is not clear, however, that all of any needed increase would be reflected in an equal brightening of the ZAMS, because there are now recognized to be compensating line-blanketing factors involved in fitting other clusters' main sequences to the Hyades main sequence (van den Bergh 1977*b*; Turner 1979*b*). In fact, the part of the ZAMS that includes B stars (on which distances to the Cepheid clusters mainly depend) is probably essentially correct as it stands, even though it was originally tied to the Hyades distance modulus (Johnson 1963; Blaauw 1963). The evidence for this consists of three kinds of *independent* calibrations of the ZAMS for B stars, based on (1) trigonometric parallaxes of nearby A and F field stars to establish the zero point (Johnson and Iriarte 1958; Crawford 1975, 1978, 1979); (2) kinematical parallaxes of B stars in the Sco-Cen moving group (Blaauw 1963; Anthony-Twarog 1982), although a systematic error of 0.4 mag (in the *opposite* direction from proposed revisions to the Hyades distance modulus) is still possible (Jones 1970); and (3) the lower envelope of absolute magnitudes determined for the B type components of eclipsing binary systems (Olson 1968). The third method (probably the most reliable method) can now be improved by using the new radii and effective temperatures compiled by Popper (1980) for B type stars in detached binary systems. Popper's deduced absolute magnitudes are plotted in Figure 2, where the lower envelope agrees excellently with the standard Johnson-Blaauw ZAMS, which is also plotted for comparison. For these reasons we may with some justification regard the cluster Cepheid absolute magnitudes in Tables 1 and 2 as being essentially *independent* of the Hyades distance modulus.

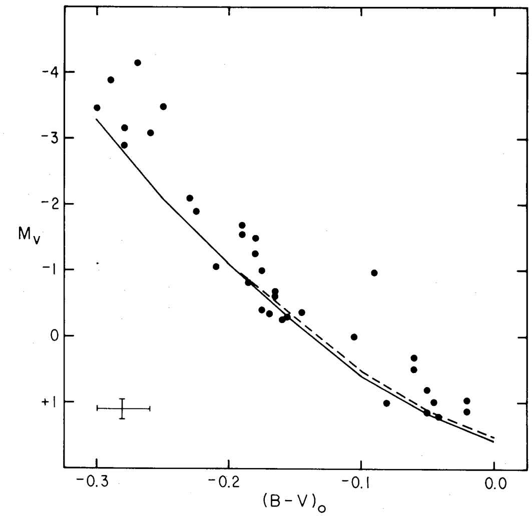

FIG. 2.—Color-magnitude diagram for B type components of detached eclipsing-binary systems in the solar neighborhood. Data are from Popper; the mean error bar illustrates the uncertainty. Also shown is the standard ZAMS for B stars: *dashed line*, Johnson calibration; *continuous line*, Blaauw calibration. This shows that the standard ZAMS for B stars is probably correct.

A few of the Cepheid cluster distances have also been determined with Hβ photometry, calibrated by the use of nearby trigonometric-parallax stars (Crawford 1978; Eggen 1974). With respect to the distance moduli used in the present paper, Eggen (1980, 1982) finds little change for NGC 6087, the VY Car group, and the RZ Vel group. On the other hand, Schmidt (1980*b*, *c*, 1981, 1982*a*, *b*) has consistently obtained shortfalls, averaging 0.33 mag, in the case of NGC 6664, NGC 7790, M25, NGC 129, and NGC 6087 (note that these shortfalls go in the *opposite* direction from recent revisions to the Hyades distance modulus). Such disturbing differences provide a quantitative estimate of the possible size of accidental and systematic errors that inevitably creep into any determination of a cluster distance modulus (Anthony-Twarog 1982). The mean error of ± 0.18 attached to our derived value for $\langle M_V \rangle(0.8)$ is only an internal mean error that *might* be less than the true error.

Fortunately, however, independent methods of deriving $\langle M_V \rangle(0.8)$ exist. Despite the large mean errors, they yield consistent results. An unweighted mean for each method listed in Table 1 is -3.53, -3.8, -3.66, and -3.63. The unweighted mean of all 15 determinations in the table is

$$\langle M_V \rangle(0.8) = -3.62 \pm 0.15 \text{ (est. m.e.)} .$$

Agreement among the different methods suggests that interstellar reddening and interstellar extinction have been properly allowed for in all cases.

De Vaucouleurs (1978*a*) has derived a similar mean value, $\langle M_V \rangle(0.8) = -3.60 \pm 0.15$, but for a number of reasons the agreement must be considered accidental. First of all, only four determinations were used to obtain his mean. Second, he adopted only one statistical-parallax result, Jung's (1970) very discrepant value, which happens to compensate almost exactly the very bright cluster-fitting value that he obtained by increasing the cluster Cepheid luminosities by 0.26 mag in order to accomodate an upward revision of the Hyades distance modulus. Third, he applied two additional corrections to Sandage and Tammann's (1969) absolute magnitudes for cluster Cepheids, one of which (the color dependence of the total extinction) is negligible and the other (the color dependence of the ratio R_V of total to selective extinction) should not have been applied. As Sandage and Tammann (1969) pointed out, cluster Cepheid absolute magnitudes are *independent* of R_V if the total extinction A_V of the Cepheids is taken to be the same as that of the cluster B stars.

b) RR Lyrae Stars

To limit the possible scatter of RR Lyrae luminosities due to variations in chemical composition, only values of $\langle M_V \rangle$ determined for "almost pure" samples of *metal-poor* (Preston's $\Delta S > 5$) RR Lyrae stars will be used here. Recently published determinations of $\langle M_V \rangle$ are listed in Table 3. No correction for period has been applied, because in globular clusters the short-period (Bailey type *c*) variables have apparent magnitudes that differ insignificantly ($\Delta\langle M_V \rangle = -0.01 \pm 0.01$ mag) from the apparent magnitudes of the long-period (Bailey type *ab*) variables (Sandage 1981; Sandage, Katem, and Sandage 1981). Although mean parallaxes for Bailey type *c* variables belonging to the field have often indicated the possibility that these stars have much fainter absolute magnitudes (van Herk 1965; Woolley *et al.* 1965; Heck 1972, 1973; Hemenway 1975), the samples used in these studies were seriously contaminated by what would now be classified as dwarf Cepheids and δ Scuti stars, which have clearly brought down the luminosities.

In Table 3, two entries (those for Oke *et al.* and Woolley *et al.*) are averages of results obtained in different papers by the same authors. We have not included one very discordant result based on the modified Baade-Wesselink method, $\langle M_V \rangle = -0.3$ (Balona 1978) which Balona himself distrusted, and one semiempirical value, $\langle M_V \rangle = 0.7$, based on fitting globular-cluster main sequences to theoretically computed isochrones (Carney 1980). Also not included are results based on main-sequence fits to a deblanketed Hyades main sequence; only direct fits to a main sequence defined by nearby metal-poor subdwarfs with meaningful trigonometric parallaxes are sufficiently reliable to be included here. Finally, we have omitted Graham's (1979) result for RR Lyrae stars in the Large and Small Magellanic Clouds because it depends, in part, on the classical Cepheid luminosity calibration.

Average values of $\langle M_V \rangle$ for the four different methods listed in Table 3 are 0.67, 0.60, 0.51, and 0.62, respectively. The scatter is very small. All twenty determinations together yield

$$\langle M_V \rangle(\mathrm{RR}) = 0.61 \pm 0.15 \text{ (est. m.e.)} .$$

De Vaucouleurs (1978*a*) obtained $\langle M_V \rangle(\mathrm{RR}) = 0.80 \pm 0.15$ using only the statistical-parallax method (omitting Woolley and Savage's 1971 result). But the statistical-parallax method is at present an unreliable approach, in view of the markedly different results that have been obtained from Greenwich proper-motion data,

TABLE 3

OBSERVATIONAL DETERMINATIONS OF $\langle M_V \rangle$ FOR METAL-POOR RR LYRAE STARS

Method	$\langle M_V \rangle$	N	References
Secular or statistical parallax	0.82 ± 0.22[a]	164	van Herk 1965
	0.52 ± 0.30	119	Woolley *et al.* 1965
	0.60 ± 0.21	19	Woolley and Savage 1971
	0.88 ± 0.30	59	Clube and Jones 1971; Jones 1973
	0.6 ± 0.3	50	Heck 1972 (Clube and Jones data)
	0.51 ± 0.20	134	Heck 1972, 1973, 1975
	0.49 ± 0.42	172	Hemenway 1975
	0.63 ± 0.30	124	Heck and Lakaye 1978
	0.7 ± 0.2	~60	Clube and Dawe 1978 (Heck data)
	1.00 ± 0.25	59	Clube and Dawe 1978, 1980
Moving group	0.60 ± 0.13	5	Eggen and Sandage 1959
Modified Baade-Wesselink	0.43 ± 0.37	3	Oke and Bonsack 1960; Oke, Giver, and Searle 1962; Oke 1966
	0.46 ± 0.23	8	Woolley and Dean 1976; Woolley and Davis 1977
	0.57 ± 0.20	34	Kraft 1977 (McDonald data)
	0.6 ± 0.2	1	Wallerstein and Brugel 1979
	0.60 ± 0.20	2	Manduca *et al.* 1981
	0.42 ± 0.07	3	Siegel 1982
Cluster main-sequence fitting	0.6 ± 0.2	...	Sandage 1970
	0.40 ± 0.20	...	Hanson 1979
	0.87 ± 0.16	...	Sandage 1982
Unweighted mean	0.61	...	

[a] Using $\langle M \rangle = (M_{\max} + M_{\min})/2 + 0.14$ (Oort and Plaut 1975).

$\langle M_V \rangle \approx 1.0$, and from Liège data, $\langle M_V \rangle \approx 0.6$ (Heck and Lakaye 1978; Clube and Dawe 1978). Since most older proper-motion data as well as three other methods of deriving $\langle M_V \rangle$ support the traditional value of about 0.6 mag, we believe that de Vaucouleurs has significantly overestimated $\langle M_V \rangle$(RR).

c) *Type II Cepheids*

The subgroup of galactic type II Cepheids known as BL Herculis stars occupies a small period range of 1–3 days and shows very little dispersion in visual absolute magnitude. In galactic globular clusters this subgroup always lies 1.04 ± 0.22 mag above the RR Lyrae stars (Rosino 1978). From cluster main-sequence fitting, we found that $\langle M_V \rangle(\mathrm{RR}) = 0.62 \pm 0.20$ (Table 3). Therefore, we have

$$\langle M_V \rangle(\mathrm{Cep\ II}) = -0.42 \pm 0.29 \text{ (in clusters)} .$$

Independent calibrations of the luminosities of these variables are at present limited to two stars, both of which have been studied by the modified Baade-Wesselink method: the metal-poor field variable XX Vir, with $\langle M_V \rangle = -0.7 \pm 0.3$ (Wallerstein and Brugel 1979), and the metal-rich field variable BL Her itself, with $\langle M_V \rangle = -0.4 \pm 0.4$ (Abt and Hardie 1960). The mean of these two values is

$$\langle M_V \rangle(\mathrm{Cep\ II}) = -0.55 \pm 0.25 \text{ (in the field)} .$$

This confirms that field and cluster BL Her stars have very similar absolute magnitudes. An average of the two mean values is

$$\langle M_V \rangle(\mathrm{Cep\ II}) = -0.49 \pm 0.19 .$$

Problems with using short-period type II Cepheids as standard candles are their great rarity and their easy confusion with two brighter classes: the so-called *anomalous* type II Cepheids and the short-period *classical* Cepheids. To the author's knowledge no *normal* type II Cepheid of short period has yet been discovered outside the Galaxy. Although less confusion exists for the very rare long-period type II Cepheids (W Virginis stars and RV Tauri stars), these variables do not follow a tight period-luminosity relation (Rosino 1978).

d) *Variables in the Magellanic Clouds*

Classical Cepheids and RR Lyrae stars are found together as members of the Large and Small Magellanic Clouds. Therefore the *difference* between the luminosities of these two classes of variable stars can be established very accurately.

In the Large Cloud, if we use published period-luminosity relations, the classical Cepheids have $\langle V \rangle(0.8) = 14.87$ (Woolley *et al.* 1962), 15.12 (Gascoigne 1969), 15.38 (Hodge and Wright 1969), 15.20 (Butler 1978), 15.13 (Gascoigne and Shobbrook 1978), and 15.15 (Martin, Warren, and Feast 1979); a straight average of these values is $\langle V \rangle(0.8) = 15.14 \pm 0.07$. Similarly, in the Small Cloud, $\langle V \rangle(0.8) = 15.41$ (Arp 1960, as corrected by van Genderen 1969), 15.42 (Gascoigne 1969), and 15.44 (Butler 1976), leading to an average value of $\langle V \rangle(0.8) = 15.42 \pm 0.01$. Reddening within the Clouds must be very small for these Cepheids of short period, which avoid dusty regions and show remarkably blue colors. In clear regions of the Clouds, Crampton (1979) and Crampton and Greasley (1982) have found $E_{B-V} = 0.03 \pm 0.01$ for both Clouds, which agrees closely with what de Vaucouleurs (1978*b*, 1980) and many other authors (McNamara and Feltz 1980, Tables 1 and 2) have found. An *internal* extinction correction of $A_V = 3.3E_{B-V} = 0.10 \pm 0.03$ will therefore be adopted.

RR Lyrae members of the Large Cloud have $\langle V \rangle(\mathrm{RR}) = 19.20$, both in the general field near NGC 1783 (Graham 1977) and inside the cluster NGC 2257 (Gascoigne 1966; Cowley and Hartwick 1981). In the Small Cloud, $\langle V \rangle(\mathrm{RR}) = 19.57$ in the general field around NGC 121 (Graham 1975) and 19.45 within NGC 121 (Tifft 1963), leading to a mean of $\langle V \rangle(\mathrm{RR}) = 19.51 \pm 0.06$. Internal extinction can be ignored for these regions that lie very far from the Cloud centers.

Assembling the foregoing results, we obtain the following differences:

$$\text{LMC:} \quad \langle V_0 \rangle(0.8) - \langle V_0 \rangle(\mathrm{RR}) = -4.16 \pm 0.07 ,$$

$$\text{SMC:} \quad \langle V_0 \rangle(0.8) - \langle V_0 \rangle(\mathrm{RR}) = -4.19 \pm 0.07 .$$

For comparison, we have also:

$$\text{Galaxy:} \quad \langle M_V \rangle(0.8) - \langle M_V \rangle(\mathrm{RR}) = -4.23 \pm 0.21 .$$

The similarity of these numbers is remarkable in view of the fact that the classical Cepheid populations diverge considerably in regard to metal content, mean color, and frequency distribution of periods. Thus it would appear that the mean period-luminosity relation for classical Cepheids is truly universal, as Gascoigne (1974) previously suspected on other grounds. This makes the relation an exceptionally accurate tool for extragalactic research.

III. THEORETICAL CALIBRATIONS

Stellar models are now sufficiently advanced in sophistication that one can begin to rely on them to calibrate the extragalactic distance scale, especially as large errors continue to plague the purely observational methods. A large body of nonlinear full-amplitude models for RR Lyrae stars and for type I and type II Cepheids has quite recently been constructed for the express purpose of deriving accurate masses and luminosities (Vemury and Stothers 1978; Carson, Stothers, and Vemury 1981; Stothers 1981; Carson and Stothers 1982, 1983; Hubickyj 1983). These models employ the Carson opacities, which have led to a better prediction of light-curve (and velocity-curve) characteristics and implied masses for the Population II variables than have the available versions of the Los Alamos opacities; therefore the luminosity predictions of these models deserve some consideration. Nevertheless, it is reassuring that the Los Alamos opacities give model results that are at least reasonable, even if not very precise (Christy 1966; Hodson, Cox, and King

1980). For classical Cepheids, however, both sets of opacities lead to unsatisfactory and sometimes contradictory predictions, as we shall show in § IV, although the situation seems to be more serious in the case of the Los Alamos opacities (Christy 1968; Stobie 1969*a*, *b*; Castor *et al.* 1976; Simon and Davis 1983).

A few preliminary words about the Carson opacities seem necessary. These opacities differ rather subtly from the Los Alamos opacities in the temperature regime important to Cepheid-type pulsation (see Fig. 4 of Stothers 1981). Therefore the stability characteristics of the models, which depend on the *gradient* of opacity, are affected considerably more than the inertial properties like periods. On the other hand, one need not worry about the possible effects of a large (and controversial) CNO opacity bump, which appears only in Carson's calculations, because this bump occurs at temperatures too high to be important for Cepheid-type pulsation.

A model envelope for a Cepheid-like variable is uniquely determined by specifying the star's mass M, luminosity L, effective temperature T_e (or radius R), and chemical composition (X, Y, Z). Since the models under consideration were calculated with the same input physics and the same computer code, they possess the great advantage of homogeneity, a valuable asset for intercomparison of different classes of models. For each class, computed light curves (and velocity curves) were compared with selected observed curves to obtain the best possible matches. From this comparison, masses and luminosities of the variables could be inferred, no other observational data being needed, since the period automatically fixes the effective temperature if the mass and luminosity are specified. Over the whole range of models we find

$$P \approx 0.022(R/R_\odot)^{7/4}(M/M_\odot)^{-3/4} \text{ days} , \qquad (2)$$

where $R = (L/4\pi\sigma T_e^4)^{1/2}$ and P is the fundamental-mode period. Because the results proved to be relatively insensitive to chemical composition, the main surveys were conducted with assigned compositions of $(X, Y, Z) = (0.739, 0.240, 0.021)$ for the classical Cepheid models and $(X, Y, Z) = (0.745, 0.250, 0.005)$ for the RR Lyrae and type II Cepheid models.

Normalization and transformation quantities were the following: $L_\odot = 3.90 \times 10^{33}$ ergs s^{-1}; $M_{\text{bol}\,\odot} = 4.75$; BC and $(B-V)_0$ as functions of effective temperature, from Flower (1977). Flower's empirical bolometric corrections are very close to the theoretical values computed by Buser and Kurucz (1978) and by Bell and Gustafsson (1978), who also found little dependence on metallicity. With very little uncertainty, we may set BC = 0.00 for RR Lyrae stars, short-period type II Cepheids, and 6 day classical Cepheids.

a) *Classical Cepheid Models*

Ten classical Cepheid models have been matched successfully with *individual* observed Cepheids (Carson and Stothers 1983). To reduce the scatter arising from the small number of stars involved, normal points can be formed from neighboring pairs of models having nearly the same period; the results are collected in Table 4. A least squares fit to the absolute magnitudes gives $\langle M_V \rangle = -2.24(\pm 0.21) - 2.60(\pm 0.16) \log P$. The formal discrepancy in slope between this relation and equation (1) hardly seems significant, and so we shall adopt the more accurately determined empirical slope and solve again for the theoretical zero point, obtaining

$$\langle M_V \rangle = -1.73(\pm 0.05) - 3.01 \log P . \qquad (3)$$

The standard deviation for a single normal point is ± 0.12 mag. Equation (3) then yields

$$\langle M_V \rangle (0.8) = -4.14 \pm 0.12 .$$

b) *RR Lyrae Models*

In the same manner as for the classical Cepheids, it has been found that $\log (L/L_\odot) = 1.65 \pm 0.05$ for metal-poor RR Lyrae field stars of both Bailey types *ab* (Stothers 1981) and Bailey type *c* (Hubickyj 1983), i.e.,

$$\langle M_V \rangle (\text{RR}) = 0.62 \pm 0.13 .$$

Inferred masses for both Bailey types are $0.60 \pm 0.05\, M_\odot$.

An independent method of deriving $\langle M_V \rangle$(RR) was proposed a few years ago by Christy (1966). From theoretical model calculations, it was discovered that a transition line on the H-R diagram separated RR Lyrae models that pulsated in either the fundamental mode or the first overtone from cooler models that pulsated only in the fundamental mode. Expressed in terms of the transition period P_{tr} (in days), Christy's result was $L/L_\odot = (P_{\text{tr}}/0.057)^{1.67}$. Christy identified the transition line observationally as the boundary between type *c* (first-overtone) pulsators and type *ab* (fundamental-mode) pulsators. Stellingwerf (1975) later computed a quite different transition line, and, to add to the uncertainty, Spangenberg (1975) as well as Cox (1980*a*) suggested that the Christy transition line may not even

TABLE 4

NORMAL POINTS FOR OBSERVATIONALLY MATCHED CLASSICAL CEPHEID MODELS

P (days)	$\log (L/L_\odot)$	$\log T_e$	$\log (R/R_\odot)$	$M/M_\odot$	$\langle M_V \rangle$	$(B-V)_0$	N
8	3.70 ± 0.04	3.81	1.76	6.0 ± 0.5	-4.60 ± 0.10	0.43	2
14	3.93 ± 0.07	3.77	1.95	8.5 ± 0.5	-5.12 ± 0.18	0.59	2
17	4.08 ± 0.07	3.76	2.05	9.5 ± 0.5	-5.48 ± 0.18	0.63	2
23	4.23 ± 0.07	3.76	2.12	10.5 ± 0.5	-5.85 ± 0.18	0.63	2
35	4.37 ± 0.07	3.75	2.22	11.5 ± 0.5	-6.21 ± 0.18	0.67	2

exist at RR Lyrae luminosities. Using the recent discoveries of double-mode RR Lyrae stars in globular clusters, Cox, Hodson, and Clancy (1983) have proposed that the fundamental blue edge of the instability strip is probably the true dividing line between type *c* and type *ab* pulsators. With masses derived from the periods of the double-mode pulsators in the manner of Jorgensen and Petersen (1967) and with the transition periods also known, Cox, Hodson, and Clancy were able to infer, from the period-mass-radius relation, $\langle M_V \rangle = 0.60 \pm 0.10$ (internal m.e.) for the variables in M3 (Oosterhoff group I) and $\langle M_V \rangle = 0.30 \pm 0.10$ (internal m.e.) for the variables in M15 (Oosterhoff group II). The average of these two values is $\langle M_V \rangle = 0.45 \pm 0.07$. Because these are linear-theory results, it might be expected that the replacement of the Los Alamos opacities adopted in Cox, Hodson, and Clancy's work by the Carson opacities would yield a similar mean absolute magnitude; this turns out to be the case, since the Carson opacities give $\langle M_V \rangle = 0.32 \pm 0.07$ (Hubickyj 1983). On account of possibly large systematic errors arising from such a complicated chain of analysis, these absolute magnitudes should not be regarded as significantly different from the mean value derived above directly from the light curves.

The apparent division of the luminosities by Oosterhoff group, however, does agree in sign with Sandage's (1982) suggestion of a 0.2 mag difference between Oosterhoff groups I and II among globular clusters. On the other hand, moving-group parallaxes (Eggen and Sandage 1959) and new (but not old) statistical parallaxes (Heck and Lakaye 1978; Clube and Dawe 1978) suggest a division having the *opposite* sign, while the Baade-Wesselink method and the horizontal-branch stars in the globular cluster ω Cen (Kraft 1977) suggest nearly equal luminosities for the two Oosterhoff groups.

c) *Type II Cepheid Models*

For normal type II Cepheids with 1–2 day periods, the light-curve method of deriving luminosities gives $\log (L/L_\odot) = 2.08 \pm 0.08$, or

$$\langle M_V \rangle(\text{Cep II}) = -0.45 \pm 0.20$$

(Carson, Stothers, and Vemury 1981; Carson and Stothers 1982). The standard deviation is large because the comparison was made with stars having a wide range of metal abundances. Inferred masses of these stars are $0.60 \pm 0.05\ M_\odot$—in good agreement with masses derived for the metal-poor RR Lyrae stars. An average mass of $\sim 0.60\ M_\odot$ for Population II variables also agrees with much other observational and theoretical data (see the papers cited above).

IV. DISCUSSION

Five independent methods (four of them observational and one theoretical have been used in the present paper to determine the visual absolute magnitudes of classical Cepheids and RR Lyrae stars; three of the methods have also been used here for short-period type II Cepheids. The observationally determined absolute magnitudes for all three classes of stars have met an external consistency test, which compares their relative values in the field with their relative values in the Large Magellanic Cloud, Small Magellanic Cloud, and galactic globular clusters, as shown by Table 5. A third test, illustrated by Figure 3, compares their observationally derived absolute magnitudes with their theoretically predicted ones.

Only one result falls outside the small range of possible error; namely, the theoretically predicted absolute magnitudes of classical Cepheids are too bright by 0.5 mag (4 σ discrepancy). One reason for the failure of the classical Cepheid models, in the face of the considerable success enjoyed by the RR Lyrae and type II Cepheid models, is possibly a *nonuniqueness* (or near nonuniqueness) of the predicted light curves and velocity curves for classical Cepheids. In contrast to the models for the other variables, classical Cepheid models can occupy a significant range of mass and luminosity at any effective temperature. Since the present classical Cepheid models are also too *blue* compared to observations (Dean, Warren, and Cousins 1978), a practical test of the hypothesis of nonuniqueness is to look at some fainter and redder models. We point out a possible model for the star X Cyg ($P = 16$ days) with assigned theoretical parameters $\log (L/L_\odot) = 3.70(\langle M_V \rangle = -4.42)$, $\log T_e = 3.72$, and $M/M_\odot = 7$ (Vemury and Stothers 1978), which agrees nearly as well (except for its light amplitude) with the observed star as does the much brighter model selected by Carson and Stothers (1983). It is noteworthy

TABLE 5

SUMMARY OF ABSOLUTE MAGNITUDE DATA FOR CEPHEIDS AND RR LYRAE STARS

		$\langle M_V \rangle$ OR $\Delta\langle M_V \rangle$	
STARS	OBSERVATIONAL METHOD USED	Observed	Theoretical
RR*abc* (low metals)	Table 3 methods	$+0.61 \pm 0.15$	$+0.62 \pm 0.13$
Cep II ($\log P < 0.5$)	Modified Baade-Wesselink	-0.55 ± 0.25	-0.45 ± 0.20
Cep I ($\log P = 0.8$)	Table 1 methods	-3.62 ± 0.15	-4.14 ± 0.12
RR*c* *minus* RR*ab*	Galactic globular clusters	-0.01 ± 0.01	0.00 ± 0.18
Cep II *minus* RR*ab*	Galactic globular clusters	-1.04 ± 0.22	-1.07 ± 0.24
	Absolute determinations (above)	-1.16 ± 0.29	-1.07 ± 0.24
Cep I *minus* RR*ab*	Magellanic Clouds	-4.18 ± 0.05	-4.76 ± 0.18
	Absolute determinations (above)	-4.23 ± 0.21	-4.76 ± 0.18

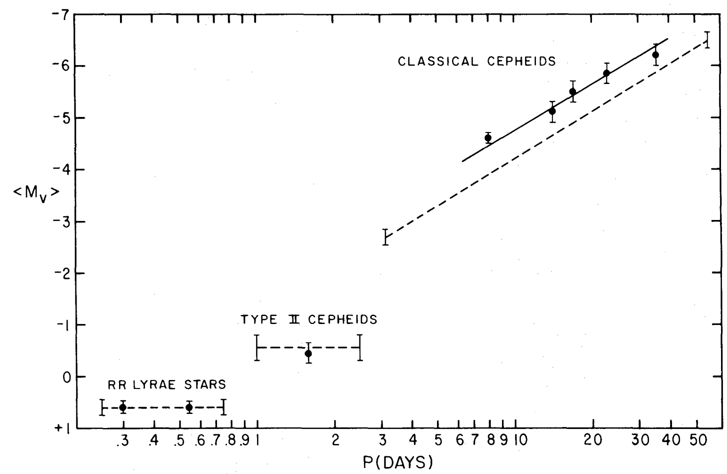

FIG. 3.—Mean period-luminosity relation for three classes of galactic variable stars. *Dashed lines* refer to observed stars; *dots* and the *continuous line* refer to theoretical models.

that the fainter model lies *below* the observed period-luminosity relation shown in Figure 3. On the other hand, both the fainter and brighter models obey the same normal evolutionary mass-luminosity relation! This problem of possible nonuniqueness of the theoretical light and velocity curves is compounded by the fact that most of the normal models, except in a few isolated cases, do not adequately represent the Hertzsprung sequence of observed light and velocity curves of classical Cepheids. The latter problem (usually expressed as a "mass discrepancy") affects all current calculations of normal classical Cepheid models (Cox 1980*b*).

For these reasons, we must give zero weight to the theoretically predicted luminosities of classical Cepheids. Four observational methods, however, have already established that

$$\langle M_V \rangle(0.8) = -3.62 \pm 0.15 .$$

Fitted to a slope of $d\langle M_V \rangle/d \log P = -3.01$, this zero-point luminosity leads to

$$\langle M_V \rangle = -1.21 - 3.01 \log P . \tag{4}$$

In the case of the metal-poor RR Lyrae stars, we shall adopt an average of the observationally determined value of $\langle M_V \rangle$(RR) (based on four methods) and the theoretically determined one (with a weight of one-fifth):

$$\langle M_V \rangle(\mathrm{RR}) = 0.61 \pm 0.15 .$$

For normal short-period type II Cepheids, two observational methods and one theoretical method (all equally weighted) give

$$\langle M_V \rangle(\mathrm{Cep\ II}) = -0.47 \pm 0.15 .$$

If the theoretical results are not used, the derived absolute magnitudes of RR Lyrae stars and short-period type II Cepheids are not sensibly changed.

The estimated mean errors of ± 0.15 mag come mainly from an examination of the scatter of the absolute magnitudes among *all* the determinations used for each class of variable star. A better estimate can be derived from the scatter among the *means* of the determinations based on the various methods used for each class of variable star. In that case the error is only ± 0.10 mag. (By their nature, these error estimates include both accidental and systematic errors; but one important source of possible systematic error has been studiously

TABLE 6

OBSERVATIONAL DETERMINATIONS OF DISTANCE TO THE GALACTIC CENTER

Method	Object	R_0 (kpc)	References
Counts with distance	Main-sequence stars[a]	9.0 ± 2	van den Bergh 1974
	RR Lyrae stars	8.7 ± 0.6	Oort and Plaut 1975
	Globular clusters[b]	8.5 ± 1.0	Harris 1976, 1980
Galactic rotation	Supergiants[c]	8.0	Taff and Littleton 1972
	OB stars and Cepheids	8.8 ± 1.1	Thackeray 1972
	OB stars	9.0 ± 2	Balona and Feast 1974
	H II regions[c]	9.0	Crampton and Georgelin 1975
	OB stars	8.0 ± 1	Crampton *et al.* 1976
Unweighted mean	All objects	8.6	

[a] Independent calibration.
[b] Same calibration as for RR Lyrae stars.
[c] Same calibration as for OB stars.

avoided in this paper: the uncertain Hyades cluster distance modulus.) In practice, the actual determination of galactic and extragalactic distances will not reach such a high level of accuracy, because of bias in the objects selected (the zero point rests on averages), uncertain interstellar extinction corrections, and various other difficulties in fitting objects to a distance scale, no matter how accurately the zero point may be determined (Baade 1963).

It will for the present be illuminating and useful to apply our results to two immediate problems: first, the distances to the Large and Small Magellanic Clouds and, second, the distance to the center of our Galaxy.

If we adopt for the Magellanic Clouds a space-dependent internal extinction correction from § II*d* and a uniform foreground extinction correction of $A_V = 3.3E_{B-V} = 0.10 \pm 0.03$ based on a foreground reddening of $E_{B-V} = 0.03 \pm 0.01$ (McNamara and Feltz 1980; Crampton 1979; Crampton and Greasley 1982), we obtain true distance moduli as follows: for the Large Cloud, $(m - M)_0 = 18.56 \pm 0.17$ (classical Cepheids) or 18.49 ± 0.15 (RR Lyrae stars); for the Small Cloud, $(m - M)_0 = 18.84 \pm 0.16$ (classical Cepheids) or 18.80 ± 0.16 (RR Lyrae stars). As might have been anticipated, classical Cepheids and RR Lyrae stars give nearly identical distance moduli; the resulting mean distance modulus for each Cloud is

$$\text{LMC:}\quad (m - M)_0 = 18.52 \pm 0.11\ ,$$

$$\text{SMC:}\quad (m - M)_0 = 18.82 \pm 0.11\ .$$

These moduli correspond to distances of 51 kpc and 58 kpc, respectively. For comparison, if the same extinction corrections are adopted as above, then spectroscopic parallaxes of bright main-sequence OB stars in the Clouds give, with a rather low precision, $(m - M)_0 = 18.6 \pm 0.2$ (LMC) (Crampton 1979) and $(m - M)_0 = 19.1 \pm 0.3$ (SMC) (Crampton and Greasley 1982). Using a number of miscellaneous distance indicators, de Vaucouleurs (1978*b*, 1980) derived moduli which are smaller by ~ 0.2 mag than ours, primarily because he adopted more foreground extinction.

Lastly, we can check the consistency of the classical Cepheid and RR Lyrae distance scales by comparing a number of recent determinations of the Sun's distance from the galactic center that have been based on (1) standard absolute magnitudes for OB stars and (2) $\langle M_V \rangle \approx 0.6$ for RR Lyrae stars. These determinations are listed in Table 6. With certain adjustments, other determinations could be added: for example, $R_0 = 9.0 \pm 2$ kpc, from van den Bergh's (1971) counts of RR Lyrae stars corrected to $\langle M_V \rangle = 0.6$; and $R_0 = 8.0$ kpc, from Rosino's (1978) tabulation of distances to globular clusters using various $\langle M_V \rangle$(RR) values. One could also base R_0 on counts of long-period variables, if their absolute magnitudes were securely enough known. Yet despite the differences of the methods used in Table 6 and the uncertainties of the extinction corrections, all the values tabulated lie in the narrow range $R_0 = 8$–9 kpc. An unweighted mean of the eight values is $R_0 = 8.6 \pm 0.5$ (est. m.e.) kpc.

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